

MULTIMEDIA



UNIVERSITY

STUDENT ID NO

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MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2017/2018

TMA 1301 – COMPUTATIONAL METHODS

(All sections / Groups)

7 MARCH 2018  
2.30 p.m – 4.30 p.m  
(2 Hours)

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**INSTRUCTIONS TO STUDENTS**

1. This Question paper consists of **5 pages** only with **4 Questions**.
2. Attempt **ALL FOUR** questions. All questions carry equal marks and the distribution of the marks for each question is given.
3. Please write your answers in the Answer Booklet provided, and **start each solution of a question on a new page**.
4. Show all steps.

**Question 1**

- (a) Given the function  $f(x) = x \left( \sqrt{x + \frac{1}{x}} - \sqrt{x - \frac{1}{x}} \right)$
- (i) Calculate the value of  $f(10)$  by using **six-digit arithmetic** with rounding. [1 mark]
  - (ii) Convert the given function into a form that avoids the **loss of significance**. [2 marks]
  - (iii) Calculate the value of  $f(10)$  using the function obtained in part (ii) and **six-digit arithmetic** with rounding. [1 mark]
  - (iv) Taking 0.31623 as an actual value for  $f(10)$ , find the **absolute errors** for the computed value from part (i) and (iii). [1 mark]
- (b) (i) Let  $f(x) = x^4 - 7x^3 + 18x^2 - 20x + 8$ . Is **Newton's Method** converges quadratically or linearly to the root  $r = 2$  for the given  $f(x)$ ? Then find  $\lim_{n \rightarrow \infty} \frac{e_{n+1}}{e_n^2}$  (if converges quadratically) or  $\lim_{n \rightarrow \infty} \frac{e_{n+1}}{e_n}$  (if converges linearly), where  $e_n$  denotes the error at step  $n$ . [1.5 marks]
- (ii) Suppose  $f(x)$  is a function on interval  $[1, 2]$  with  $f(x) = x^2 + 4x - 10$ . Perform **three** iterations using **Newton's method** to reach an approximated root of the equation  $f(x) = 0$ , starting with initial value  $p_0 = 1$ . Use **FIVE decimal places** and **show** all the working steps. [3.5 marks]

Continued.....

**Question 2**

[Note: For this question, use **FOUR decimal places** for all the workings.]

- (a) Find the actual value for the definite integral  $\int_0^2 3^x dx$ .

$$[\text{Note: } \int a^x dx = \frac{a^x}{\ln a} + C]$$

[1 mark]

- (b) Approximate  $\int_0^2 3^x dx$  by using the **Trapezoidal Rule** with 5 points, and hence find the **absolute error**.

[4 marks]

- (c) Approximate  $\int_0^2 3^x dx$  by completing the following table using **Romberg algorithm**, and hence, find the **absolute error**.

	m=0	m=1	m=2
n=0	$R(0,0)=$ _____		
n=1	$R(1,0)=$ _____	$R(1,1)=$ _____	
n=2	$R(2,0)=$ _____	$R(2,1)=$ _____	$R(2,2)=$ _____

[4 marks]

- (d) Comment on the accuracy of integral obtained in part (b) and (c).

[1 mark]

Continued.....

**Question 3**

(a) Consider the following linear system:

$$\begin{bmatrix} 2 & 3 & 1 \\ 4 & 1 & 4 \\ 3 & 4 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 9 \\ \frac{7}{2} \end{bmatrix}$$

- (i) Use row reduction technique to find an upper triangular **U** and lower triangular **L** in the **LU factorization** of the given linear system.
- (ii) Then find the values of  $x_1$ ,  $x_2$  and  $x_3$  by using the **L** and **U** obtained from part (i).

[5.5 marks]

(b) Construct the equations for  $x$ ,  $y$  and  $z$  of the following linear system. Then compute the two iterations for  $x$ ,  $y$  and  $z$  using the **Gauss-Seidel Method**

$$\begin{aligned} 4x + y - z &= 3 \\ 2x + 7y + z &= 19 \\ x - 3y + 12z &= 31 \end{aligned}$$

**Copy** the following table into your **Answer Booklet** and **complete** it. Write your answers correct to **FOUR** decimal places.

$n$	$x$	$y$	$z$
0	0	0	0
1			
2			

[2.5 marks]

(c) Find the **eigenvalues** for the following **matrix A**:

$$A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$$

[2 marks]

**Continued.....**

**Question 4**

- (a) The following table shows the consumer price index (CPI) of Malaysia for second half year of 2016.

Month (x)	July	August	September	October	November	December
CPI (y)	115.1	115.6	115.3	115.7	116.9	116.6

The CPI of Malaysia for second half year of 2016 is assumed can be fitted using *linear least squares*.

[Note: Round your answers to **ONE** decimal place.]

- (i) Copy the following table into your Answer Booklet and complete it.

Month	$x$	$y$	$x^2$	$xy$
July	1	115.1		
August	2	115.6		
September	3	115.3		
October	4	115.7		
November	5	116.9		
December	6	116.6		
	$\sum x =$	$\sum y =$	$\sum x^2 =$	$\sum xy =$

[2 marks]

- (ii) From (i), find the equation of the best fit linear line  $y = a + bx$  using the *least squares method*.

[3.5 marks]

- (iii) From (ii), estimate the CPI of Malaysia in February of year 2017.

[0.5 mark]

Continued.....

(b) Given the following *divided difference* (DD) table of a function  $f$ .

$x_k$	$y_k$	First DD	Second DD	Third DD
0				
		-1		
2	-2		$\frac{3}{2}$	
4	8			
		-9		
6	-10			

[Note: For (i) and (ii), give your answers in the SIMPLEST FRACTION form.]

(i) Complete the above table.

[2 marks]

(ii) Find the *cubic Newton polynomial*  $P_3(x)$ .

[1.5 marks]

(iii) Approximate  $f(1)$  from the obtained  $P_3(x)$  in (ii).

[0.5 mark]

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